## Laboratory 2 (drawn from lab textby Actatrore)

## Instrument Familiarization and Basic Electrical Relations

## Required Components:

- $21 \mathrm{k} \Omega$ resistors
- $21 \mathrm{M} \Omega$ resistors
- $12 \mathrm{k} \Omega$ resistor


## Objectives

This exercise is designed to acquaint you with the following laboratory instruments which will be used throughout the semester:

- The Oscilloscope
- The Digital Multimeter (DMM)
- The Triple Output DC power Supply
- The AC Function Generator

During the course of this laboratory exercise you should also obtain a thorough working knowledge of the following electrical relations:

- Series and Parallel Equivalent Resistance
- Kirchoffs Current Law (KCL)
- Kirchoff s Voltage Law (KVL)
- Ohm's Law
- The Voltage Divider Rule
- The Current Divider Rule

The experiments to be performed during this laboratory are also designed to introduce you to two very important instrument characteristics:

- The output impedance of a real source
- The input impedance of a real instrument

A thorough explanation of the proper use of each of the instruments above will be presented when you come to the laboratory. You should already be familiar with the basic electrical relations listed above; however, a quick review will follow.

## Series and Parallel Equivalent Resistance

It can be shown that when resistors are connected in series the equivalent resistance is the sum of the individual resistances

$$
\begin{equation*}
R_{e q}=R_{1}+R_{2}+\cdots+R_{N} \tag{2.1}
\end{equation*}
$$



Figure 2.1 Series Resistors
For resistors connected in parallel,

$$
\begin{equation*}
\frac{1}{R_{e q}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\cdots+\frac{1}{R_{N}} \tag{2.2}
\end{equation*}
$$



Figure 2.2 Parallel Resistors.

For two resistors in parallel, Equation 2.2 can be written as:

$$
R_{e q}=\frac{R_{1} R_{2}}{R_{1+} R_{2}}
$$

## Kirchoff s Voltage Law (KVL)

Kirchoff $s$ Voltage Law (KVL) states that the sum of the voltages around any closed loop must equal zero:

$$
\sum_{i=1}^{N} V_{i}=0
$$

For example, applying KVL (starting at point A) to the circuit shown in Figure 2.3 gives:

$$
-V+V_{1}+V_{2}=0
$$

or

$$
V=V_{1}+V_{2}
$$



Figure 2.3 Kirchoff s Voltage Law

## Kirchoff s Current Law (KCL)

Kirchoffs Current Law (KCL) states that the sum of the currents entering (positive) and leaving (negative) a node must equal zero:

$$
\sum_{i=1}^{N} I_{i}=0
$$

For example, applying KCL to the circuit shown in Figure 2.4 gives:

$$
I-I_{1}-I_{2}=0
$$

or

$$
I=I_{1}+I_{2}
$$



Figure 2.4 Kirchoffs Current Law
Ohm's Law
Ohm's Law states that the voltage across an element is equal to the resistance of the element times the current through it:

$$
\begin{equation*}
\mathrm{V}=\mathrm{IR} \tag{2.10}
\end{equation*}
$$



Figure 2.5 Ohm's Law
The Voltage Divider Rule
The voltage divider rule is an extension of Ohm's Law and can be applied to a series resistor circuit shown in Figure 2.6.


Figure 2.6 Voltage Division
The current flowing in the circuit is

$$
I=\frac{V}{R_{e q}}=\frac{V}{R_{1}+R_{2}}
$$

Applying, Ohm's Law, the voltage across $R_{2}$ is

$$
V_{o}=I R_{2}
$$

Thus the voltage divider relation is

$$
V_{o}=V\left(\frac{R_{2}}{R_{1}+R_{2}}\right)
$$

## The Current Divider Rule

The current divider rule is can be derived by applying Ohm's Law to the parallel resistor circuit shown in Figure 2.7.


Figure 2.7 Current Division

The current flowing from the voltage supply is:

$$
I=\frac{V}{R_{e q}}=\frac{V\left(R_{1}+R_{2}\right)}{R_{1} R_{2}}
$$

Applying Kirchoffs Voltage Law around the outside loop gives:

$$
V=I_{2} R_{2}
$$

Combining the two equations above gives:

$$
I=\frac{I_{2}\left(R_{1}+R_{2}\right)}{R_{1}}
$$

Solving for $\mathrm{I}_{2}$ gives the current divider relation:

$$
I_{2}=I \frac{R_{1}}{R_{1}+R_{2}}
$$

## Root-Mean-Square Values

When dealing with AC signals, voltage and current values can be specified by their root-mean-square (rms) values. An rms value is defined as the square root of the average of the square of a signal integrated over one period. For current and voltage, the rms relations are

$$
I_{R M S}=\sqrt{\frac{1}{T} \int_{0}^{T} I^{2} d t}=\frac{I_{m}}{\sqrt{2}} \text { and } V_{R M S}=\sqrt{\frac{1}{T} \int_{0}^{T} V^{2} d t}=\frac{V_{m}}{\sqrt{2}}
$$

where $I_{m}$ and $V_{m}$ are the amplitudes of sinusoidal current and voltage waveforms. Rms values are useful for power calculations. For example, the average AC power dissipated by a resistor can be calculated with the same equations that are used with DC signals:

$$
P_{\text {avg }}=V_{R M S} I_{R M S}=R I_{R M S}^{2}=\frac{V_{R M S}^{2}}{R}
$$

## Real Sources and Meters

When analyzing electrical circuits on paper the concepts of ideal sources and meters are often used. An ideal voltage source has zero output impedance and can supply infinite current. An ideal voltmeter has infinite input impedance and draws no current. An ideal ammeter has zero input impedance and no voltage drop across it. Laboratory sources and meters have terminal characteristics that are somewhat different from the ideal cases. The terminal characteristics of the
real sources and meters you will be using in the laboratory may be modeled using ideal sources and meters as illustrated in Figures 2.8 through 2.10.


Figure 2.8 Real Voltage Source with Output Impedence


Figure 2.9 Real Ammeter with Input Impedence


Figure 2.9 Real Voltmeter with Input Impedence.
In some instances as you will see, the input impedance of a meter or the output impedance of a source can be neglected and very little error will result. However, in many applications where the impedances of the instruments are of a similar magnitude to those of the circuit serious errors will occur.

As an example of the effect of input impedance, if you use an oscilloscope or multimeter to measure the voltage across $\mathrm{R}_{2}$ in Figure 2.6, the equivalent circuit is:


Figure 2.11 Effect of Input Impedence
The equivalent resistance of the parallel combination of $R_{2}$ and $R$, is:

$$
\begin{equation*}
R_{e q}=\frac{R_{2} R_{i}}{R_{2}+R_{i}} \tag{2.20}
\end{equation*}
$$

Therefore, the actual measured voltage would be:

$$
\begin{equation*}
V_{o}=\frac{R_{e q}}{R_{1}+R_{e q}} V_{i} \tag{2.21}
\end{equation*}
$$

If $\mathrm{R}_{\mathrm{i}}$ is large compared to $\mathrm{R}_{2}$ (which is usually the case), $R_{e q} \approx R_{2}$ and the measured voltage ( $\mathrm{V}_{0}$ ) would be close to the expected ideal voltage division result of $\frac{R_{2}}{R_{1}+R_{2}} V_{i}$. However, if $R_{2}$ is not small compared to $R_{i}$, the measured voltage will differ from the ideal result based on above Equations.

If you know values for $V_{i}, R_{1}$, and $R_{2}$ in Figure 2.11, and measure $V_{0}$, you can determine the input impedance $\left(R_{i}\right)$ of the measuring device using the following analysis. Equation 2.21 can be solved for $R_{\text {eq }}$ giving:

$$
R_{e q}=\left(\frac{V_{o}}{V_{i}-V_{o}}\right) R_{1}
$$

Knowing $R_{\text {eq }}$, we can determine the input impedance by solving for $R_{i}$ in Equation 2.20:

$$
R_{i}=\frac{R_{e q} R_{2}}{R_{2}-R_{e q}}
$$

## Laboratory Procedure / Summary Sheet

Group: $\qquad$ Names: $\qquad$

1. Select five separate resistors whose nominal values are listed below. Record the band colors for each resistor in the table below. Then connect each resistor to the multimeter using alligator clips and record the measured value for each resistor.

| Resistor | Band Colors | Measured Value ( $\boldsymbol{\Omega} \mathbf{)}$ |
| :--- | :--- | :--- |
| $\boldsymbol{R}_{\mathbf{1}}: 1 \mathrm{k} \Omega$ |  |  |
| $\mathbf{R}_{\mathbf{2}}: 1 \mathrm{k} \Omega$ |  |  |
| $\mathbf{R}_{\mathbf{3}}: 2 \mathrm{k} \Omega$ |  |  |
| $\mathbf{R}_{\mathbf{4}}: 1 \mathrm{M} \Omega$ |  |  |
| $\mathbf{R}_{\mathbf{5}}: 1 \mathrm{M} \Omega$ |  |  |

Make sure you keep track of each of the Five resistors (e.g., by laying them out in order on the table with labels, or in the breadboard).
2. Now construct the voltage divider circuit shown using resistors $R_{1}$ and $R_{2}$ listed above and set $\mathrm{V}_{\mathrm{i}}$ to 10 Vdc using the DC power supply. When using a power supply or function generator, always adjust the supply voltages before making connections to the circuit.

Complete the table below by measuring or calculating the appropriate values. In your calculations, use the actual (measured) values for $R_{1}$ and $R_{2}$.

Note - Make sure you always have a

common ground attached to your power supply, circuit, and o-scope when taking voltage measurements with the o-scope.

Remember from Lab 1, to measure current with the multimeter, you must put the meter in series with the element of interest. So to measure the current through the resistors $R_{1}$ and $R_{2}$, you must pull out the connected ends of $R_{1}$ and $R_{2}$ and attach the meter probes between the exposed ends.

Figure 2.12 Voltage Divider Circuit


Figure 2.13 Breadboard layout for voltage divider (left) and current divider (right)

## Note - Be very careful when using the

 ammeter feature of the multimeter. If you don't place the meter in series with an element, and you put the leads across an element instead, you can burn out the meter's fuse and/or damage the device.|  | Input Voltage $\mathbf{V}_{\mathbf{i}}(\mathbf{V})$ | Output Voltage $\mathbf{V}_{\mathbf{0}}(\mathbf{V})$ | Current (mA) |
| :---: | :---: | :---: | :---: |
| Calculated | 10 V |  |  |
| Multimeter |  |  |  |
| Oscilloscope |  |  | $*$ |

*compute the current using the voltage value measured.
3. Using the oscilloscope tune the function generator to a 3 V amplitude ( 6 V peak-to-peak) sine wave at 1 kHz . Apply this voltage to the same resistor network from part 2 . Find the current and input and output voltage using the same techniques as part 2.

Complete the table below by measuring or calculating the appropriate values. In your calculations, use the actual (measured) values for $R_{1}$ and $R_{2}$. Use rms values for all table entries.
Be aware that the Lab multimeters cannot detect or measure small $\mathrm{I}_{\mathrm{RMS}}$ currents accurately.

|  | Input Voltage $\mathrm{V}_{\mathrm{i}}$ <br> (V), rms value | Output Voltage $\mathrm{V}_{\mathrm{o}}(\mathrm{V})$, <br> rms value | Current (mA), rms <br> value |
| :--- | :--- | :--- | :--- |
| Calculated | $\frac{3 V}{\sqrt{2}}$ |  |  |
| Multimeter |  |  | $*$ |
| Oscilloscope |  |  | $*$ |

*compute the current using the voltage value measured
4. Once again apply a 10 V DC voltage ( $\mathrm{V}_{\mathrm{i}}=10 \mathrm{Vdc}$ ) to the resistor network swapping $\mathrm{R}_{4}$ and $R_{5}$ in for $R_{1}$ and $R_{2}$. In this case, the impedances of the instruments are close in value to the load resistances and therefore affect the measured values. Sketch the equivalent circuit for the instruments and the attached load circuit. Use this schematic to explain differences between actual (measured) and theoretical values.

Complete the table below by measuring or calculating the appropriate values. In your calculations, use the actual (measured) values for $R_{4}$ and $R_{5}$.

|  | Input Voltage $\mathrm{V}_{\mathrm{i}}(\mathrm{V})$ | Output Voltage $\mathrm{V}_{\mathrm{o}}(\mathrm{V})$ | Current (mA) |
| :---: | :--- | :--- | :--- |
| Calculated |  |  |  |
| Multimeter |  |  |  |
| Oscilloscope |  |  | $*$ |

* compute the current using the voltage value measured

5. Construct the current divider circuit shown below using resistors $R_{1}, R_{2}$, and $R_{3}$ listed in part 1. Set the source V to 6 Vdc .


## Figure 2.14 Current Divider Circuit

Complete the table below by measuring or calculating the appropriate values. In your calculations, use the actual (measured) values for $R_{1} R_{2}$, and $R_{3}$.

|  | $\mathbf{I}_{\mathbf{1}}(\mathbf{m A})$ | $\mathbf{I}_{\mathbf{2}}(\mathbf{m A})$ | $\mathbf{I}_{\mathbf{3}}(\mathbf{m A})$ |
| :---: | ---: | ---: | ---: |
| Calculated |  |  |  |
| Multimeter |  |  |  |
| Oscilloscope | $*$ | $*$ | $*$ |

*Compute the current using the voltage values measured. To measure the voltage across $\mathrm{R}_{1}$, measure the voltages at nodes $A$ and $B$ (relative to ground) and manually subtract the values.
6. Repeat part 5 with a 3 V amplitude 500 Hz sine wave ( $\mathrm{V}=3 \sin (1000 \pi \tau)$ ).

Complete the table below by measuring or calculating the appropriate values. In your calculations, use the actual (measured) values for $R_{1}, R_{2}$, and $R_{3}$. Use rms values for all table entries.

|  | $\mathrm{I}_{1 \text { rms }}(\mathrm{mA})$ | $\mathrm{I}_{2 \text { rms }}(\mathrm{mA})$ | $\mathrm{I}_{3 \text { rms }}(\mathrm{mA})$ |
| :---: | ---: | ---: | :--- |
| Calculated |  |  |  |
| Multimeter |  |  |  |
| Oscilloscope | $*$ |  | $*$ |

* compute the current using the voltage value measured

Normally, a meter's input impedance and a source's output impedance can, to a good approximation, be neglected. In some applications, however, the impedances of the instruments are similar in magnitude to those of the circuit, causing serious errors.

## LAB 2 QUESTIONS

Names:

1. Describe how you read resistor color code values and tolerances.
2. Derive formulas, using the voltage divider and current divider rules, for the following voltage and current in Figure 2.14, using $V, R_{1}, R_{2}$, and $R 3$ only.

$$
\mathrm{V}_{1}=
$$

. $\mathrm{I}_{3}=$ $\qquad$
3. From the data collected in Part 4, calculate the input impedance of the oscilloscope and the DMM.
$Z_{\text {in }}($ scope $)=$ $\qquad$
$\mathrm{Z}_{\text {in }}(\mathrm{DMM})=$ $\qquad$

Hint: Use Equations 2.22 and 2.23. Also, if using the attenuator probe, be sure to account for the probe's impedance (see Section 3.3 in Lab 3).
4. The $A C$ wall outlet provides $110 \mathrm{~V}_{\mathrm{rms}}$ at 60 Hz . Sketch and label one period of this waveform.
5. Using a function generator and three $1 \mathrm{k} \Omega$ resistors design a circuit that will supply both a $6 \mathrm{~V} p-\mathrm{p}$ output and a 2 V p-p output. Show your work below.

